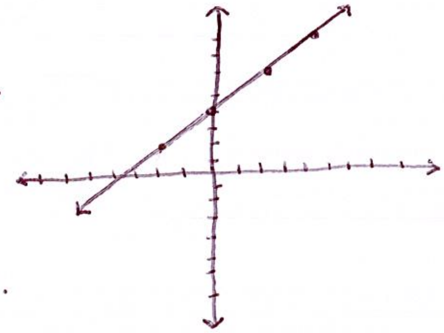
Graphs

LINEAR:

parent function: $f(x)=x \longrightarrow f(x)=\underbrace{a}_{\text{slope}}x+\underbrace{b}_{\text{y intercept}}$

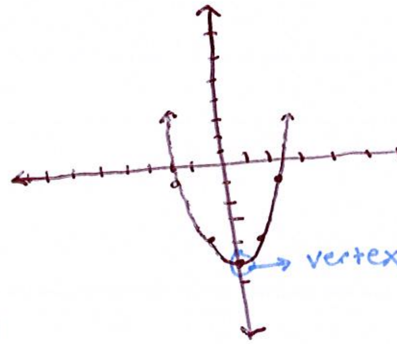


Remember:

- same slope, different y int. = parallel
- negative reciprocal slope = perpendicular
- to solve systems of equations, set the equations equal to each oter, or plug in one for x in the other.

Quadratic:

parent function: $f(x)=x^2 \longrightarrow f(x)=ax^2+bx+\underbrace{c}_{\text{y intercept}}$

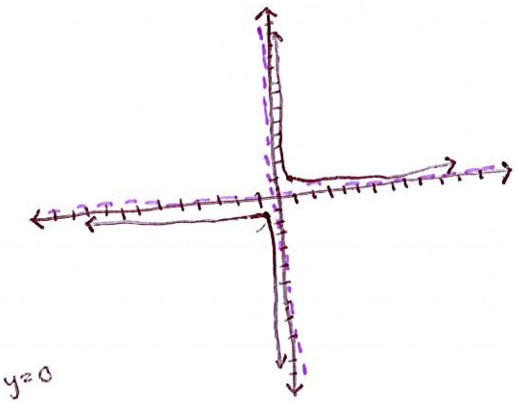


1-3-5 rule: to determine if there has been a vertical stretch or shrink, measure the slope from the vertex to the first point to its left or right. If there is no change, it will follow a pattern of 1-3-5 as it increases.

Factoring: completely factor quadratic functions to find zeroes ex. $x^2+4x+4 = (x+2)(x+2)$ } x int. = -2, with a multiplicity of 2

Reciprocal:

parent function: $f(x)=\frac{1}{x} \longrightarrow f(x)=\frac{A(\text{zeros})\cancel{(\text{hole})}}{(\text{VA})\cancel{(\text{hole})}}$

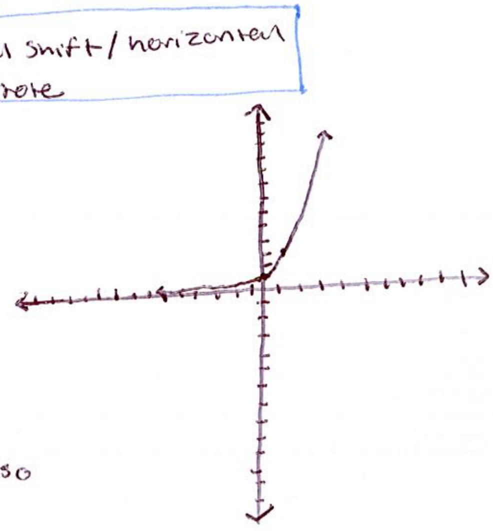


- Anything that cancels out from the numerator + denominator is a hole.
- x values that cause 0 in denominator are vertical asymptotes
- Horizontal Asymptote: if deg(num) = deg(denom), $y=\frac{a}{a}$
  - > deg(denom, n/a
  - < degree of denom, y=0
- When deg(num) = deg(denom)+1, there is a potential for slant asymptotes.
  - ↳ to find equation for slant asymptotes divide top expression by bottom expression, ignoring remainder, quotient is equation.

Exponential:

parent function: $f(x)=a^x \longrightarrow f(x)=a\cdot b^{(x-h)}+k$

- a: rate of change
- b: changes direction, $0<b<1$ = decay, $b>1$ = growth
- x: if negative, reflect over y axis
- h: horizontal shift
- k: vertical shift / horizontal asymptote



INVERSES!

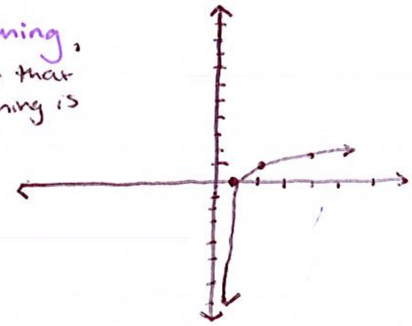
$f^{-1}(x)$ is the inverse of $f(x)$ ⇝ x and y are switched.

- inverses are symmetric about the lin $y=x$
- y int. becomes x int.
- Not all graphs pass horizontal line test (not functions) so we can restrict the domain.

LOGARITHMIC

parent function: $f(x) = \log_a x$
↳ inverse of exponential graph!
↓
If you get stuck, remember... it will follow the same pattern as exponential graphs it is just reflected.

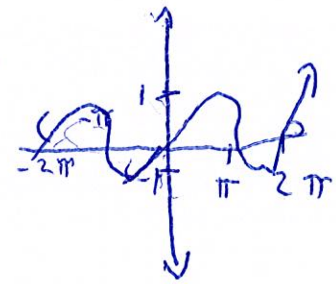
• For horizontal shrinking or stretching, multiply by $1/a$ → remember that horizontal shrinking or stretching is counter-intuitive.



Sine

Parent function: $f(x) = \sin x \rightarrow f(x) = \underset{\substack{\downarrow \\ \text{amplitude}}}{a} \sin(\underset{\substack{\downarrow \\ \frac{2\pi}{\text{period}}}}{b}(x \pm \underset{\substack{\downarrow \\ \text{phase shift}}}{ps})) + \underset{\substack{\downarrow \\ \text{vertical shift}}}{v}$

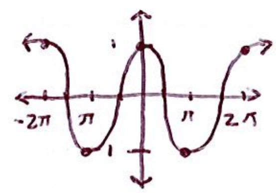
- To find period $= \frac{2\pi}{b}$
- a = amplitude.
- to find amplitudes you do $\dfrac{\text{highest value} - \text{lowest value}}{2}$
- Value for vertical shift is the new horizontal midway.



Cosine

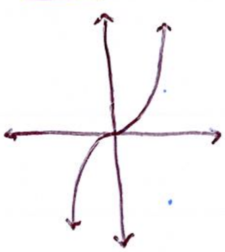
Parent function: $f(x) = \cos x \rightarrow$ follows the same basic rules as sine

- notice the same pattern as the sine graph but is at $(0,1)$.



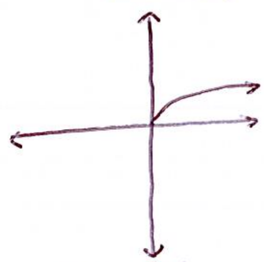
Basic patterns of others to recognize...

Cubic



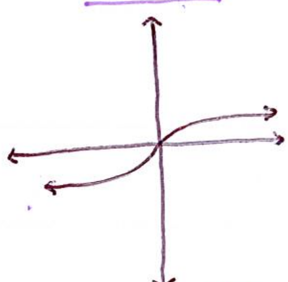
$f(x) = x^3$

Square Root



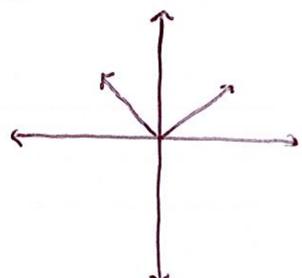
$f(x) = \sqrt{x}$

Cube Root



$f(x) = \sqrt[3]{x}$

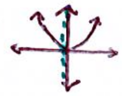
Absolute Value



$f(x) = |x|$

Even and Odd:

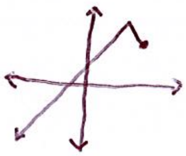
EVEN - Symmetric across the y-axis (ex. quadratic functions

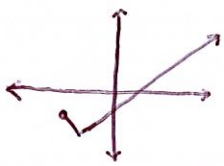


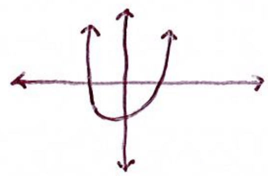
ODD - symmetric about the origin (ex. cubic functions

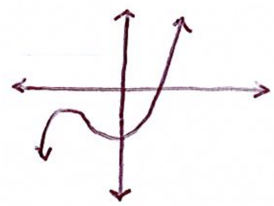
The Basics

Bounded and unbounded:









Bounded above

Bounded below

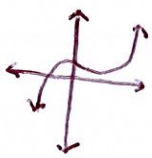
unbounded

Domain and Range:

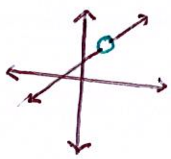
Domain → the expanse of x values included in the function
Range → the expanse of y values included in the function

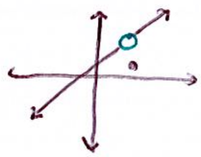
Interval notation:
parenthesis ( ) = not included
brackets [ ] = included

↳ Continuity:

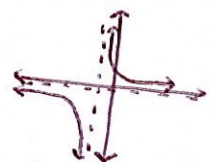


continuous

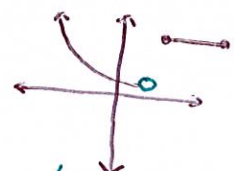




Removeable Discontinuities



(asymptote)



(jump discontinuities)

non-removeable discontinuities

End Behavior:

↳ described as the direction in which the graph is moving at each end shown in the graph.
ex.

(down, up)

Order of Transformations: Horizontal, Stretch/Shrink, Reflection, Vertical

- $f(x) \pm c$ → up/down by c
- $f(x \pm c)$ → left/right by c
- $-f(x)$ → reflect over x axis
- $f(-x)$ → reflect over y axis
- $cf(x)$ → vertical stretch/shrink
- $f(cx)$ → horizontal stretch/shrink

Turning Points:

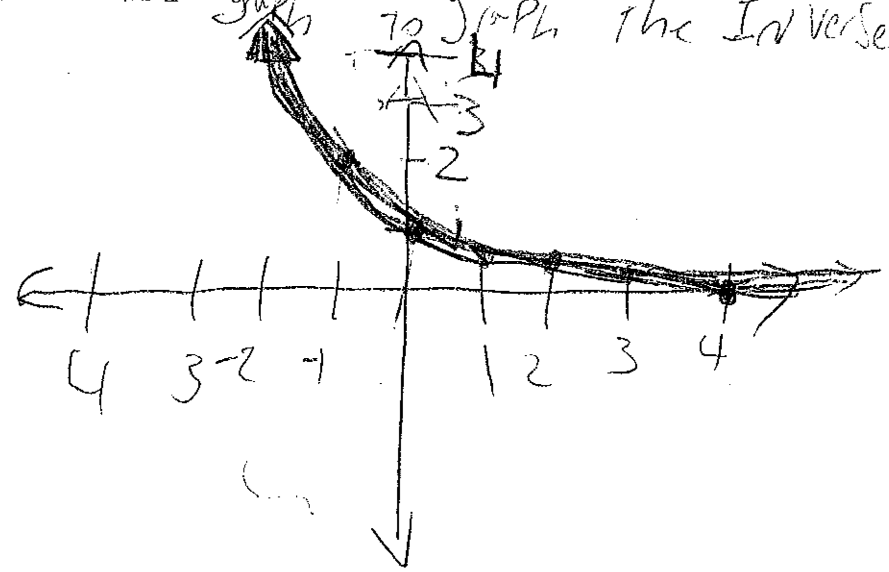
when sketching, # of turning points = degree of exponent - 1.

GRaPhs

#1: $3\cos\left(\frac{1}{3}(X + \pi)\right) + 2$

#2: Graph $X^2 - 2x - 3$ and its Inverse

#3: Use graph to graph the Inverse function



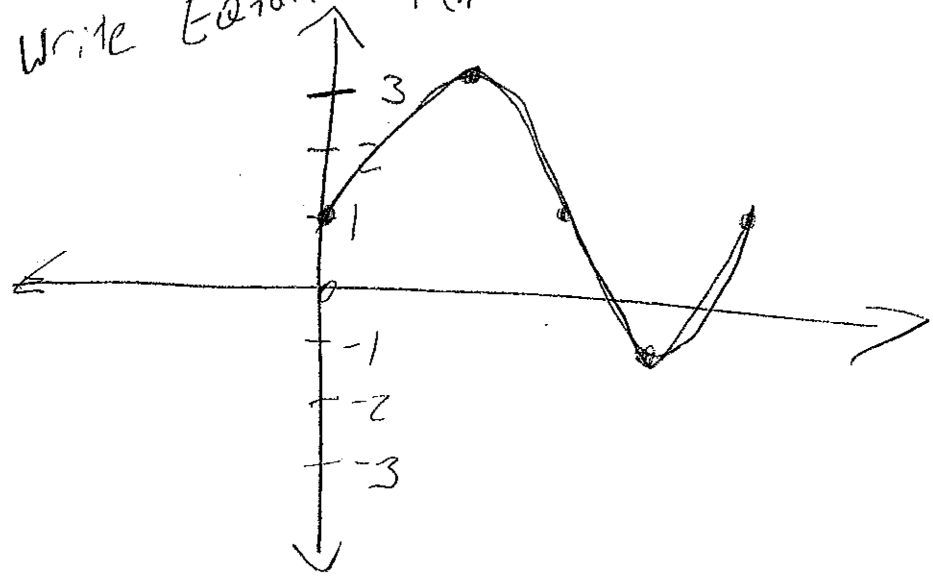
#4: Chose 2 to graph

A. $3x^2 - x^3$

B. $6x^3 - 9x - x^5$

C. $\frac{X^3}{2} - X + 2x^2 - 2$

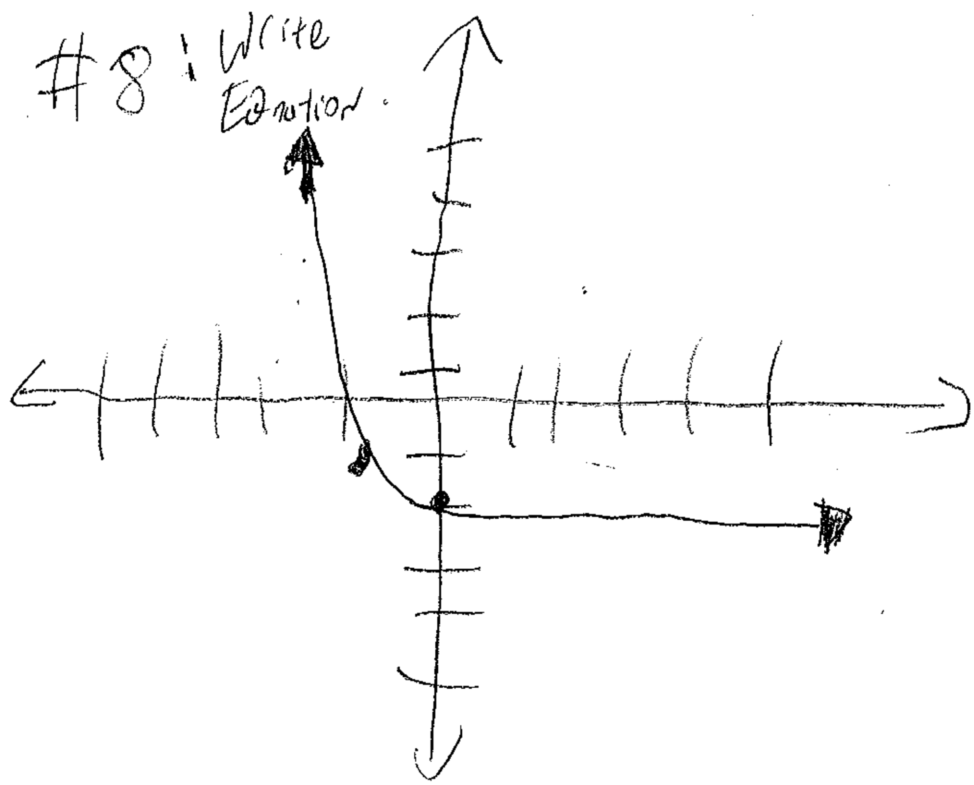
#5: Write Equation For



#6: Graph $-2^{x+1}-2$ and its: Inverse

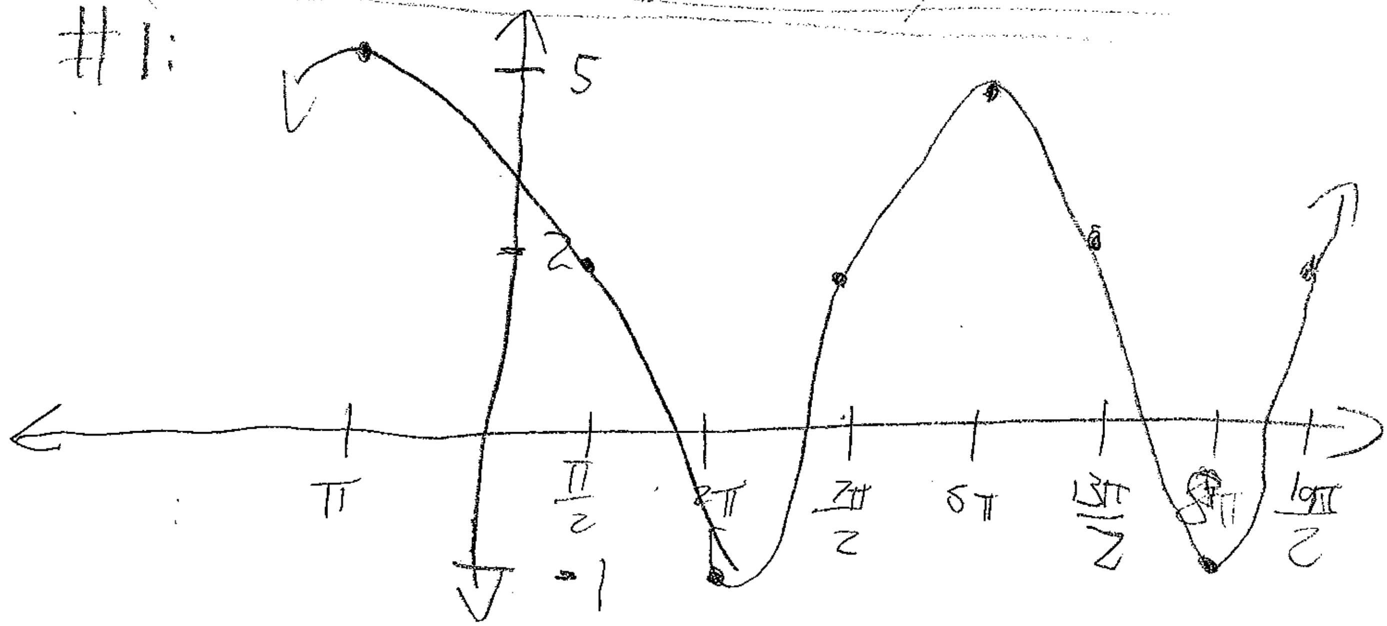
#7: graph: $-e^{x}+5$

#8: Write Equation.

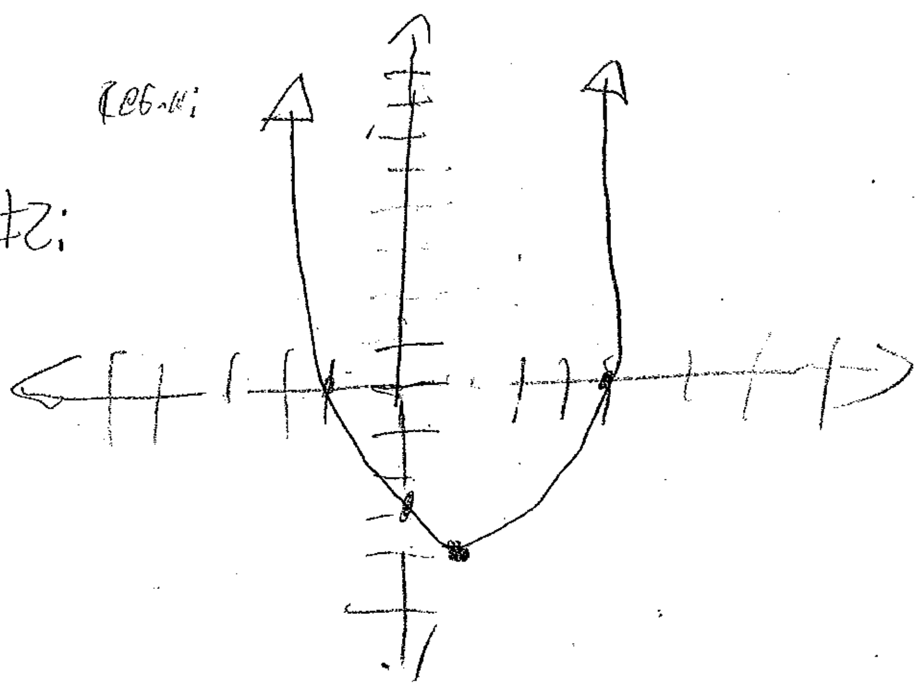


ANSWER KEY

#1:

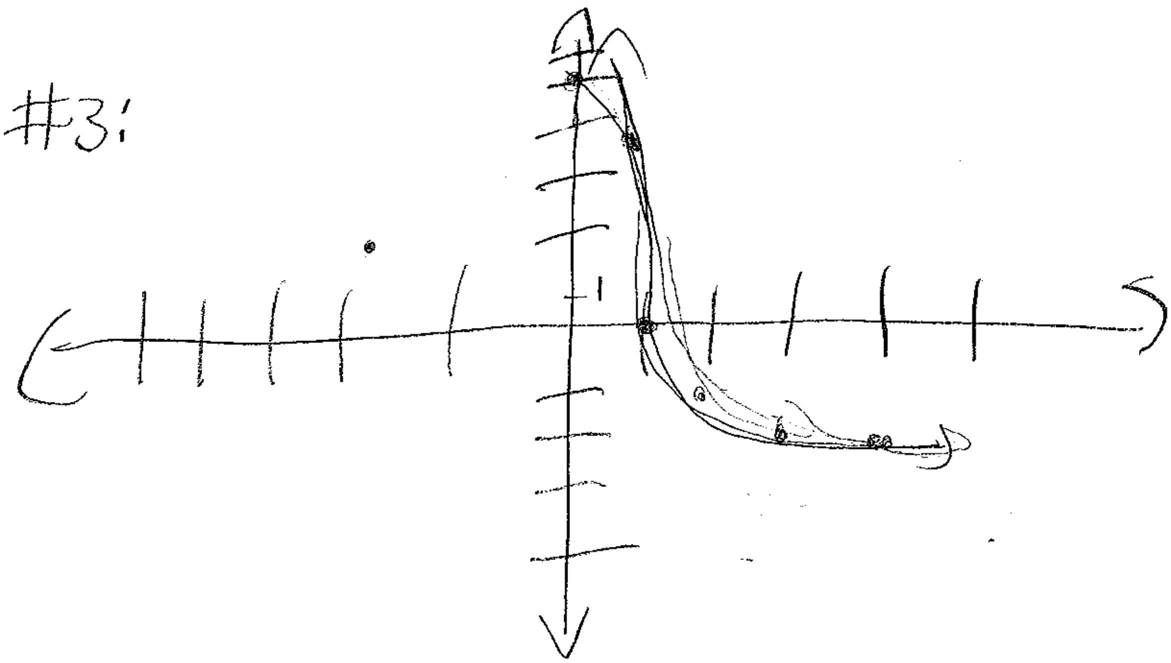


#2:



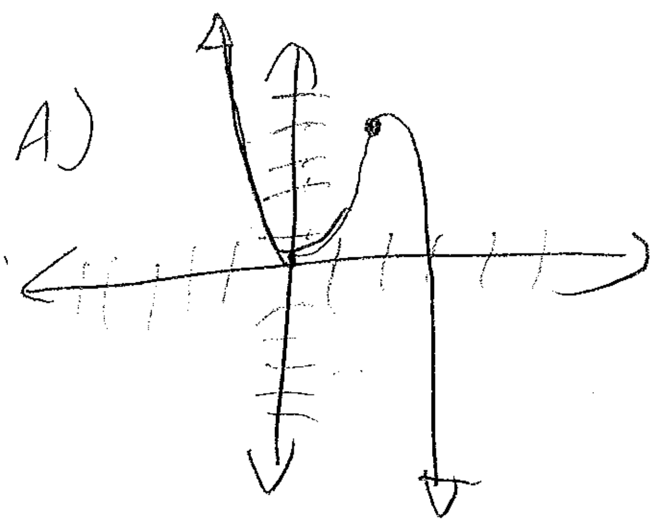
Inverse: There is no Inverse for Parabolas as it wouldn't pass the Horizontal Line test.

#3:

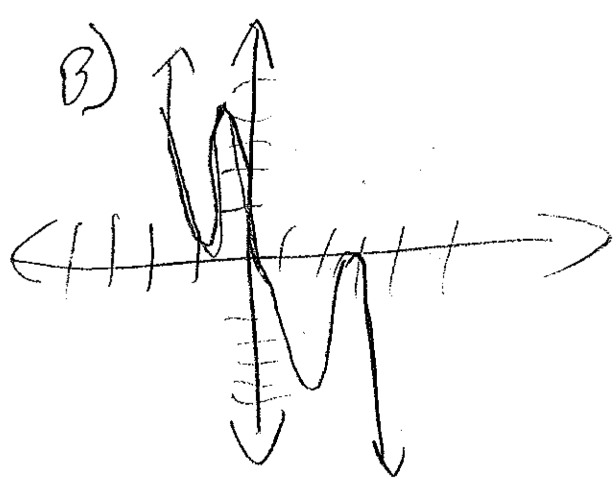


#4:

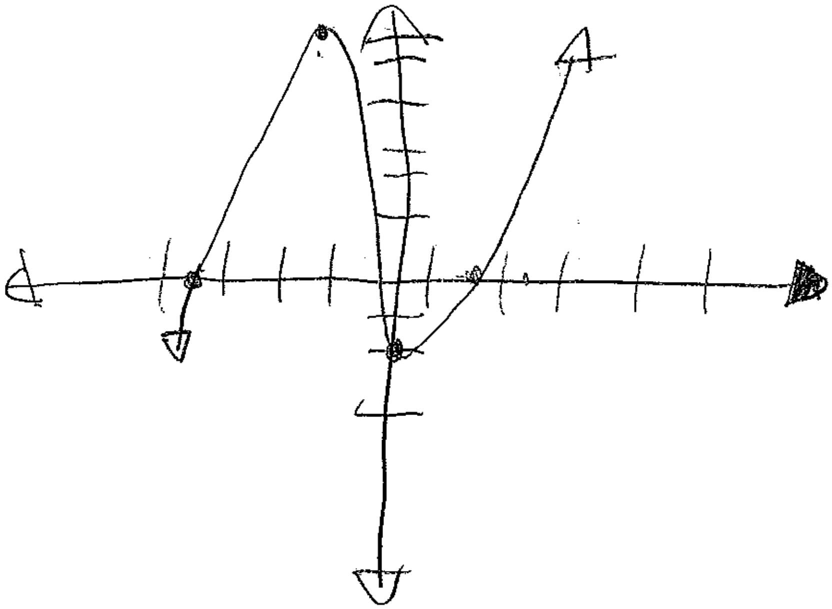
A)



B)

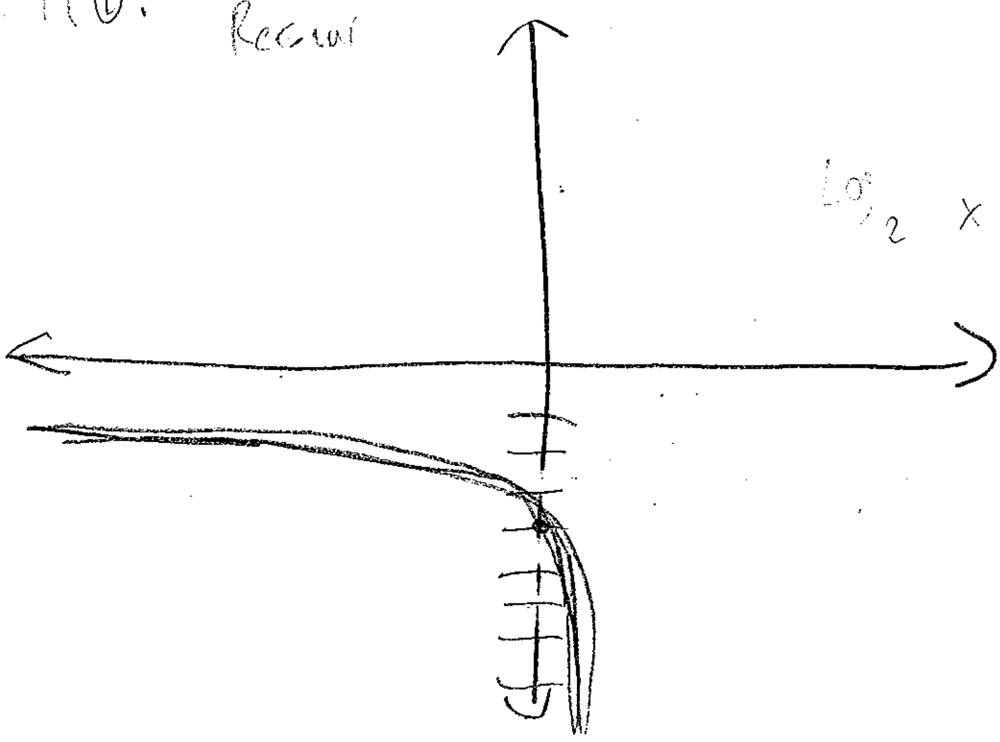


c) $\frac{x^3}{2} - x + 2x^2 - 2$



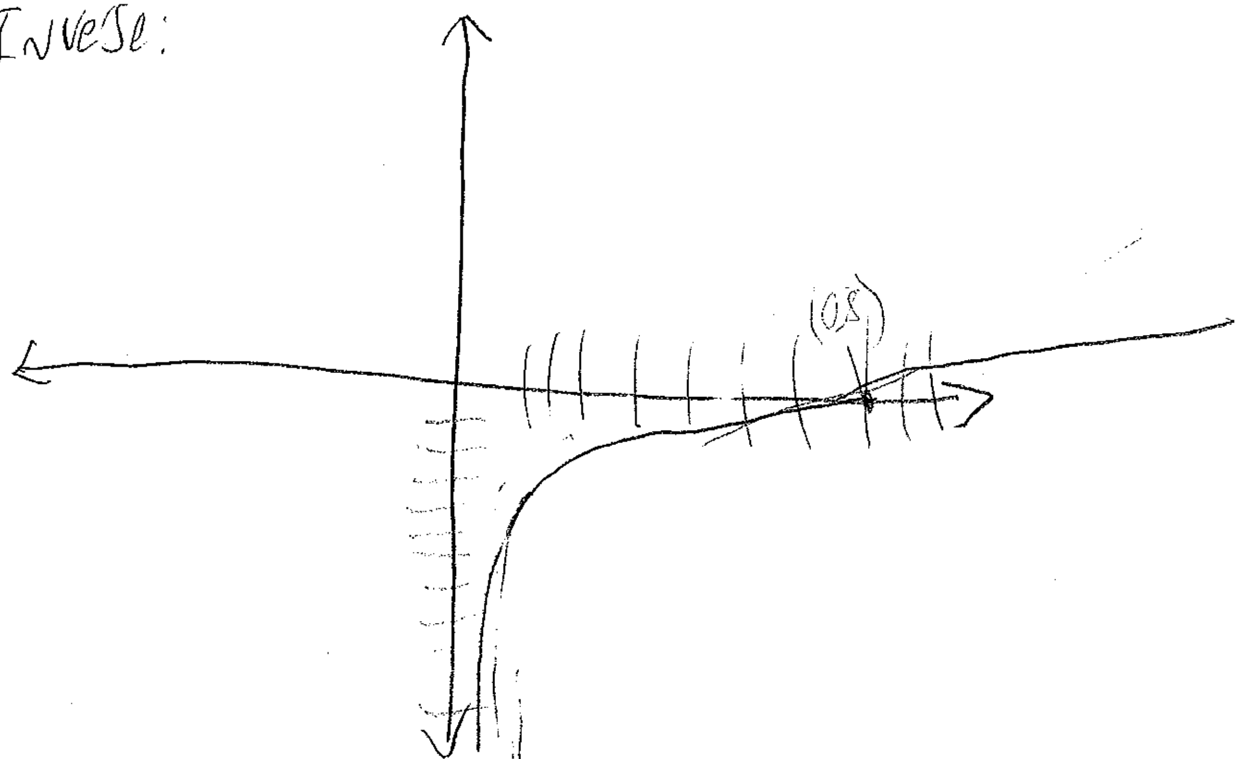
#5: $2\cos(\pi)+1$

#6: Recwi

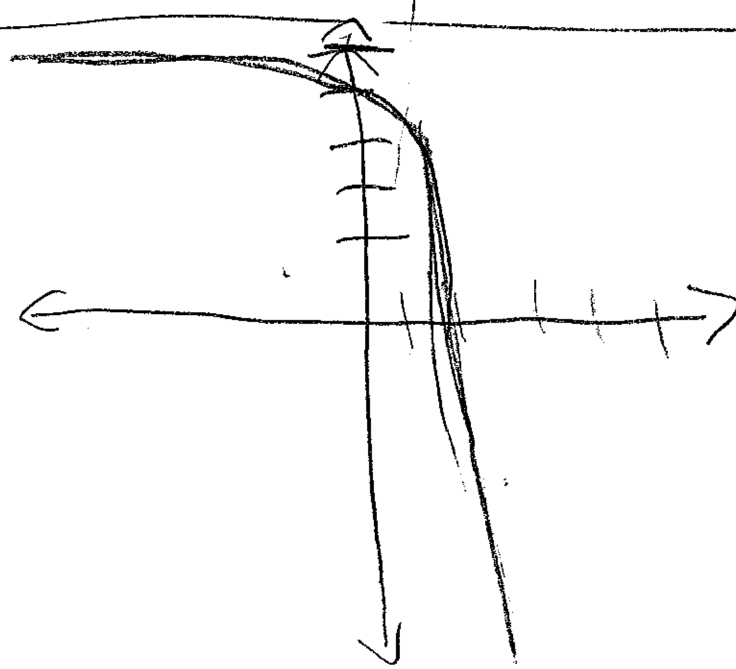


$\log_2 x+1 = y+1$

Inverse:



#7:



#8: $\frac{1}{2}^{x} - 3$